Reply by Author to T. J. Lardner

HUGH N. CHU*
North American Aviation, Inc., Canoga Park, Calif.

IN the proof of Biot's variational principle, one is led to

$$\int_v (\theta_{,i} + \lambda_{ij}H_j)\delta H_i dV + \int_{B_1} (\theta - \theta_0)\delta H_i \nu_i dS + \int_{B-B_1} \theta\delta H_j \nu_j dS = 0 \quad (1)$$

subject to the constraint $c_v\theta = -H_{i,i}$. In Eq. (1), λ_{ij} is the thermal resistivity tensor; B_1 is the boundary on which the temperature θ_0 is prescribed, whereas B-B_1 is the remaining boundary on which heat flux is prescribed; ν_i is the normal vector to the boundary; and H_i is the heat flux vector.

It follows that the assumed solution must satisfy 1) the boundary condition on temperature (which may vary with time), 2) the boundary condition on heat flux (which also may vary with time), and 3) the conservation of energy $c_v\theta = -H_{i,i}$, whereas it may approximate the Fourier's law $\theta_{,i} + \lambda_{ij}H_j = 0$.

In Ref. 1, all of these conditions are met. θ_1 is regarded as prescribed and varying with time according to Eq. (1) of Ref. 1. On the other hand, in Ref. 2 the foregoing condition 2 is replaced by the over-all heat balance equation in which heat flux through the entire surface area must be considered, including that part of the boundary on which the temperature is already specified. It seems that the general validity of the procedure of Ref. 2 requires a variational derivation.

One also must remember that, in the approximate variational methodology, there are many possible approaches to one problem. Simplicity of the procedure, as well as the accuracy, also should be considered. Equation (21) of Ref. 1 has simplicity, in that it gives the long- and short-time solutions at once without the necessity of handling two simultaneous nonlinear differential equations (which often require a digital computer, as was the case in Ref. 2).

* Senior Technical Specialist, Rocketdyne Division. Member AIAA.

References

[1] Chu, H. N., "Application of Biot's variational method to convective heating of a slab," J. Spacecraft Rockets **1**, 686–688 (1964).

[2] Lardner, T. J., "Biot's variational principle in heat conduction," AIAA J. **1**, 196–206 (1963).